3.5 Equation Solving & Modeling

Solving exponential equations

For any exponential function

$$f(x) = b^x$$

If $b^u = b^v$ then $u = v$

Ex. $3^x = 3^{2x-4}$

$x = 2x - 4$

$x = 4$

For any logarithmic function

$$f(x) = \log_b x$$

If $\log_b u = \log_b v$

then $u = v$

Ex. $\log_b 3x = \log_b 27$

$$3x = 27$$

$$x = 9$$

Ex. Solve

1) $$\frac{\cancel{20}\left(\frac{1}{2}\right)^{\frac{x}{3}}}{\cancel{20}} = \frac{5}{20}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \frac{1}{4}$$

$$\left(\frac{1}{2}\right)^{\frac{x}{3}} = \left(\frac{1}{2}\right)^{2}$$

$$\frac{x}{3} = 2$$

$$x = 6$$

Ex. Solve graphically

2) $\dfrac{e^x - e^{-x}}{2} = 5$

$Y_1 = 5$

$Y_2 = \dfrac{e^x - e^{-x}}{2}$

$x \approx 2.31\ldots$

Algebraically:

$$2e^x \cdot \frac{e^x - e^{-x}}{\cancel{2}} = 5 \cdot 2e^x$$

$$e^{2x} - e^0 = 10e^x$$

$$e^{2x} - 10e^x - 1 = 0$$, using sub. let $w = e^x$

$$w^2 - 10w - 1 = 0$$ using quad. form.

$$w = 5 + \sqrt{26} \rightarrow 5 + \sqrt{26} = e^x$$

$$5+\sqrt{26} = e^x$$

$$\ln(5+\sqrt{26}) = \ln e^x$$

$$\ln(5+\sqrt{26}) = x$$

$$2.31\ldots \approx x$$

Solving a logarithmic Equation

1) $\log x^2 = 2$ change to exp. form

$$10^2 = x^2$$

$$100 = x^2 \longrightarrow x = \pm 10$$

Ex. Solve

$$\ln(3x-2) + \ln(x-1) = 2\ln x$$

Algebraically

$$\ln(3x-2)(x-1) = \ln x^2$$

$$(3x-2)(x-1) = x^2$$

$$3x^2-5x+2 = x^2$$

$$2x^2-5x+2 = 0$$

$$(2x-1)(x-2) = 0 \qquad x = 2 \quad x \neq \tfrac{1}{2}$$

Order of magnitude

The common logarithm of a positive quantity is its order of magnitude.

comparing quantities:

1 km = 1,000 m → 1×10^3 m

So, a kilometer is 3 orders of magnitude longer than a meter.

$\$1.00 = 1 \times 10^2$

2 order of magnitude greater than one cent.

Richter Scale magnitude R of an earthquake is

$$\rightarrow \quad R = \log \frac{a}{T} + B$$

a - amplitude

Ex. How many times more severe was the earthquake in India ($R_1 = 7.9$) than the earthquake in Greece ($R_2 = 5.9$)?

$$7.9 = \log \frac{a_1}{T} + B$$

$$5.9 = \log \frac{a_2}{T} + B$$

want to find $\frac{a_1}{a_2}$

$R_1 - R_2$

$$2 = \log \frac{a_1}{T} - \log \frac{a_2}{T}$$

$$2 = \log \left(\frac{\frac{a_1}{T}}{\frac{a_2}{T}} \right)$$

$$2 = \log \frac{a_1}{a_2}$$

$$10^2 = \frac{a_1}{a_2}$$

$$100 = \frac{a_1}{a_2} \longrightarrow$$ 100 times greater

Comparing Chemical Acidity

Vinegar has PH of 2.4
Baking soda has PH 8.4

$$PH = -\log[H^+]$$

a) Find Hydrogen-ion concentration $[H^+]$

vinegar: $-\log[H^+] = 2.4$

$$\log[H^+] = -2.4$$

$$10^{-2.4} = H^+ \longrightarrow \approx 3.98 \times 10^{-3}$$

moles per liter

b) How many times greater is the hydrogen concentration of the vinegar than the baking soda?

$$\frac{[H^+]_v}{[H^+]_{BS}} = \frac{10^{-2.4}}{10^{-8.4}} = 10^6$$

times greater

c) How many orders of magnitude do the concentrations differ? 6

Newtons Law of Cooling

$$T(t) = T_m + (T_0 - T_m)e^{-kt}$$

T_m - is temp of surrounding medium

T_0 - initial temp

K - constant cooling rate

Ex. A hard boiled egg $96^\circ c$ is placed in $16^\circ c$ water to cool. Four minutes later the temp. of the egg is $45^\circ c$. When will the egg be $20^\circ c$?

$$T(t) = T_m + (T_0 - T_m)e^{kt}$$

$$45 = 16 + (96 - 16)e^{-k(4)}$$

$$45 = 16 + 80e^{-4k}$$

$$45 = 16 + 80e^{-4k}$$

$$29 = 80e^{-4k}$$

$$\frac{29}{80} = e^{-4k}$$

$$\ln\frac{29}{80} = \ln e^{-4k}$$

$$\frac{\ln\frac{29}{80}}{-4} = \frac{\cancel{-4}k}{\cancel{-4}} \rightarrow k = 0.253$$

so $\longrightarrow$

to find t when $T = 20^\circ c$

Solve:

$$20 = 16 + 80e^{(-0.253t)}$$

$$\frac{4}{80} = e^{-0.253t}$$

$$11.84 \approx \leftarrow \frac{\ln \frac{4}{80}}{-0.253} = t$$

11 min. 50 sec.